Pergamon

Int. J. Heat Mass Transfer. Vol. 41, No. 10, pp. 1129–1138, 1998

Printed in Great Britain
0017-9310/98 $19.00+0.00

PII: S0017-9310(97)00269-X

Analytical study of the effect of natural convection on cryogenic pipe freezing

A. C. KEARY* and R. J. BOWEN

Department of Mechanical Engineering, University of Southampton, Southampton SO17 1BJ, U.K.

(*Received* 12 *November* 1996 *and in final form* 4 *September* 1997)

Abstract—An analytical model of freezing a liquid inside a vertical pipe is described. Natural convection in the liquid is included by incorporating an integral solution for the heat transfer coefficient for convection driven by a heated vertical plate. The resulting model can be applied to different pipe sizes, temperatures and liquids and was used to predict the formation of an ice plug under varying freezing conditions. A method of scaling the effect of pipe diameter and initial water temperature on natural convection during freezing is proposed. The factors which limit the situations in which the model can be applied are discussed

1. INTRODUCTION

The application for this model is a technique known as pipe freezing or freeze-sealing which is used to isolate sections of a liquid filled pipeline by freezing the contents to form a solid plug. The pipe is cooled externally over a short length, typically two or three diameters long, either by immersion in a cryogen such as liquid nitrogen or by passing a coolant at a controlled temperature through a heat exchanger around the pipe. When the work on the pipe has been completed, the freezing equipment is removed and the plug allowed to thaw.

The process is simple and inexpensive compared to other plugging techniques, however, the success of a freeze is affected by a number of parameters and it is difficult to assess when the plug has blocked the pipe. In practice, freezing is carried out by contractors who rely on their experience to determine when a freeze is completed.

Any flow through the pipe will bring heat into the freezing zone and affect the potential success of the freeze. In the absence of flow, natural convection may be sufficient to transport a significant amount of heat into the region to prevent the completion of the freeze. The aim of the work described here is to develop a simple predictive model of freezing with natural convection in a vertical water filled pipe in order to be able to predict freezing times under various conditions.

2. REVIEW OF EXPERIMENTAL RESULTS

Previous experimental investigations have been carried out at Southampton to determine the effect of various parameters on the formation of ice plugs in pipes. In particular, Burton [1] and Tavner [2] studied freezing in a 100 mm diameter vertical pipe containing water. Bowen *et al.* [3] carried out a series of experiments into freezing in vertical pipes from 100 mm to 250 mm diameter, over a range of initial water temperatures.

The results obtained by Bowen *et al.* [3] for time to freeze are plotted in Fig. 1. These show that when the water is initially close to 0°C the freezing time is approximately proportional to the cross-sectional area of the pipe (i.e. proportional to the ratio of volume to surface area). As the initial temperature increases, the freezing time increases linearly due to the increasing sensible heat. As convection becomes more important, the freezing times increase more rapidly, reaching a limiting temperature above which it is not possible to obtain a complete plug. The limiting temperature is lower in larger pipes.

The shape of the plug growing inwards from the cooled wall was also observed by Burton [1] and Tavner [2]. It was found that in vertical pipes at the beginning of the freeze the plug 'neck' (where the ice layer is thickest) is skewed towards the bottom of the freezing zone due to the process of filling the jacket; after the jacket has been filled, the neck moves up the plug. In cases where there is significant natural convection, interaction between the convection and the plug formation can be observed. When the plug reaches a critical size, the fluid above the plug neck becomes thermally isolated from that below the neck and cools rapidly. This increases the freezing rate above the neck which then moves up the plug.

3. REVIEW OF PREVIOUS MODELS

The problem of freezing in a cylindrical geometry was first investigated by London and Seban [4]. They assumed one-dimensional (radial) plug growth with

* Author to whom correspondence should be addressed.

NOMENCLATURE

A_h	height of plug neck above jacket bottom as fraction of total jacket length
c	specific heat capacity
f_{δ_T}	factor for δ_T
g	acceleration due to gravity
h	heat transfer coefficient
H	jacket length
k	conductivity
L	latent heat capacity
q	ratio δ_v/δ_T
Q	heat flow
R	radius
t	time
T	temperature
x	horizontal co-ordinate; horizontal direction
z	vertical co-ordinate; vertical direction.

Greek symbols

β	thermal expansion coefficient
δ_v	velocity boundary layer thickness
δ_T	thermal boundary layer thickness
δ	difference
μ	viscosity
ρ	density.

Subscripts

b	bulk
c	coolant
i	interface
l	liquid
s	solid
T	thermal boundary layer
v	velocity boundary layer
w	wall.

Superscripts

i	inner
o	outer.

Dimensionless numbers

Grashof number $Gr_H = \beta g \Delta T \rho^2 H^3/\mu^2$

Prandtl number $Pr = \mu c_p/k$

Rayleigh number $Ra_H = \beta g c_p \Delta T \rho^2 H^3/\mu k$.

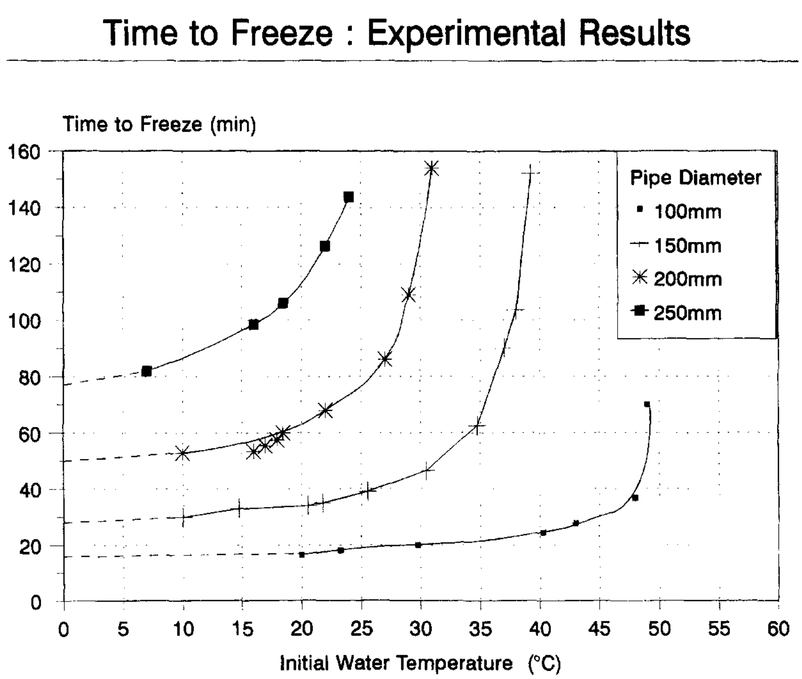

Fig. 1. Experimental results for the freezing time.

zero superheat in the liquid and neglected the change in sensible heat of the solid. An expression for the freezing rate was obtained by applying a heat balance at the freezing front. A similar approach was used by Luikov [5] and Shamsundar [6], who expanded it to incorporate a 'shape factor' to account for the twodimensional plug form. These studies differed from the method of London and Seban mainly in the choice of boundary condition on the outside of the solid region. The London and Seban method used a convective boundary condition whereas Luikov used an isothermal boundary. The resulting relationship for freezing time was proportional to the square of the pipe radius and inversely proportional to the difference between the freezing temperature and the coolant temperature.

The methods of London and Seban, Luikov, and Shamsundar were later used by Law [7]. Law applied the predictions obtained with the Luikov model to cases with initial superheat by introducing a scale factor equal to the total of the sensible and latent heats divided by the latent heat. The model of London and Seban [4] was extended by Law to include initial water superheat by assuming that the interface movement results in a change in the liquid sensible heat equal to the solidified layer changing from the initial temperature to the freezing temperature.

Lannoy [8] used a similar method and extended it to take account of forced convection by including an extra heat flux term which was formulated for both laminar and turbulent forced convection. The conditions for successful freezing were investigated for different pressure heads.

Bowen *et al.* [3] looked at the problem of freezing with natural convection and, by assuming a constant heat transfer coefficient for natural convection in the liquid, predicted that the limiting temperature in a given pipe is inversely proportional to the pipe diameter. By using available experimental data for the limiting temperature for one pipe size, the limiting temperature for any pipe size was obtained.

4. DESCRIPTION OF THE CURRENT MODEL

The problem is sketched in Fig. 2 showing the convection, conduction and solidification processes in a vertical pipe. The current method follows previous approaches in assuming no axial conduction and a steady temperature distribution in the solid phase. The plug was therefore assumed to be a hollow cylinder which gradually thickens inwards. The new aspect of this study is the inclusion of natural convection heat transfer in the liquid; this was achieved by making use of the heat transfer coefficient given by Bejan's [9] integral solution, which is described in more detail below. This approach can be used for any fluid with Prandtl number greater than 1; fluids with Prandtl number less than unity require a slightly different formulation for the heat transfer coefficient. The investigation was carried out for freezing water to form an ice plug using liquid nitrogen as the coolant to allow comparison with available experimental data.

A one-dimensional heat balance was applied at the freezing front by equating the difference between the heat flux from the water to the ice/water interface and that removed from the interface through the ice with the change in the sensible and latent heats resulting from solidification, giving equation (1).

$$2\pi R_i \rho (L + c_l(T_b - T_i)) \frac{dR_i}{dt} = Q_{conv} - Q_{cond} \quad (1)$$

The definition of heat fluxes requires knowledge of the temperature gradients on both sides of the interface; steady state solutions were used but it was noted that neglecting the effect of the moving interface means that the gradient on the solid side is over predicted. The convected and conducted heat fluxes are as follows (equations (2)).

$$Q_{conv} = 2\pi R_i h(T_b - T_i)$$

$$Q_{cond} = -\frac{2\pi(T_c - T_i)}{I + \frac{1}{k_s}\ln\frac{R_w^i}{R_i}}$$

where

$$I = \frac{1}{R_w^o h_c} + \frac{1}{k_w}\ln\frac{R_w^o}{R_w^i} \quad (2)$$

I is the thermal impedance between the ice and the nitrogen.

The velocity of the freezing front is given in equation (3).

$$\frac{dR_i}{dt} = \frac{1}{\rho(L + c_l(T_b - T_i))} \times \left[h(T_b - T_i) + \frac{T_c - T_i}{R_i\left(I + \frac{1}{k_s}\ln\frac{R_w^i}{R_i}\right)} \right] \quad (3)$$

An expression for the heat transfer coefficient was derived by Bejan [9] for natural convection driven by a heated vertical plate. This was applied to the similar situation of flow driven by a cooled plate. The temperature in the boundary layer was assumed to rise exponentially from the wall temperature to the bulk temperature (using a characteristic length δ_T); the radial velocity was zero and the vertical velocity in the boundary layer increased then decreased, given by the product of an exponential rise (length δ_v) and an exponential decay (length δ_T). These expressions were used together with the boundary later equations and solved, providing predictions of the heat transfer coefficient as well as the thickness of the velocity and thermal boundary layers and the maximum velocity. The resulting expressions are formulated in terms of the fluid properties and the length and temperature of the cooled wall. The prediction for the heat transfer coefficient is given as a function of the distance, z, from the top of the cooled wall in equation (4).

$$h(z) = \frac{k_l}{\Delta T}\frac{dT}{dx}\bigg|_{x=0} = \frac{k_l Ra^{0.25}}{f_{\delta_T} z} \quad (4)$$

where

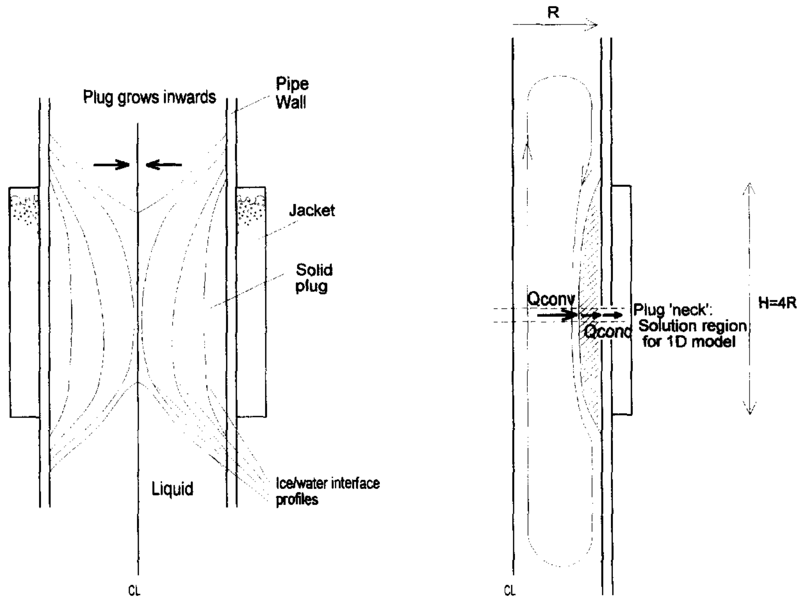

Fig. 2. Section through vertical pipe showing plug form and convection loop.

$$f_{\delta_T} = \left(\frac{8(q+0.5)(q+1)(q+2)}{3q^3}\right)^{0.25}$$

$$Pr = \frac{5}{6}q^2\frac{q+\frac{1}{2}}{q+2} \tag{5}$$

The velocity of the freezing front is therefore given in equation (6)

$$\frac{dR_i}{dt} = \frac{1}{\rho(L+c_l(T_b-T_i))}\left[\left(\frac{k_l^3\beta g c_l \rho^2 (T_b-T_i)^5}{f_{\delta_T} z\mu}\right)^{1/4} + \frac{T_c - T_i}{R_i\left(I+\frac{1}{k_s}\ln\frac{R_w^i}{R_i}\right)}\right] \tag{6}$$

The properties of water vary strongly with temperature; the heat transfer coefficient was calculated for temperatures of 10–100°C (assuming a cooled wall temperature of 0°C), using the water properties halfway between the initial temperature and 0°C. For bulk water temperatures above 10°C (away from the density inversion at 4°C), the correlation given in equation (7) fits the data to within 5%.

$$h(z) \approx 25.880 T_b^{0.680} z^{-0.25} \tag{7}$$

The value of heat transfer coefficient depends on the choice of the length, z, which determines the vertical position at which the analysis is performed. In order to calculate the freezing time, the analysis should be carried out at the plug neck. This will also be the location where the assumption of no (or negligible) axial conduction is most accurate. The distance 'z' was replaced by $A_h H$, where A_h is the proportion of the length of the freezing jacket (from the top) at the point at which the analysis is carried out.

Equations (6) and (7) were combined, together with the condition for the interface temperature for water/ice of $T_i = 0°C$, to give equation (8).

$$\frac{dR_i}{dt} = \frac{1}{\rho(L+c_l T_b)}\left[\frac{25.880 T_b^{1.68}}{(A_h H)^{0.25}} + \frac{T_c}{R_i\left(I+\frac{1}{k_s}\ln\frac{R_w^i}{R_i}\right)}\right] \tag{8}$$

The interface radius at time $(t+\delta t)$ can be estimated from the radius at time t, using equation (9).

$$R_i^{t+\delta t} \approx R_i^t + \frac{\delta t}{\rho(L+c_l T_b)}\left[\frac{25.880 T_b^{1.68}}{(A_h H)^{0.25}} + \frac{T_c}{R_i\left(I+\frac{1}{k_s}\ln\frac{R_w^i}{R_i}\right)}\right] \tag{9}$$

The variation of the conductivity of ice (k_s) (from 2.2 W/m K at 0°C to 5.0 W/m K at −196°C) was included in the model. The heat transfer between the pipe wall and nitrogen also varies with the wall temperature. When the pipe wall is relatively warm, a layer of nitrogen vapour partially insulates the wall from the nitrogen and decreases heat transfer. As the wall temperature drops, this film layer collapses, allowing contact between the nitrogen liquid and the wall, increasing heat transfer. Experimental results by Flynn [10] provide data for this behaviour; these results were incorporated into the model.

The freezing time was obtained by repeatedly updating the interface (initially set equal to the pipe radius) until the interface reached the centre of the pipe. Iteration was necessary to include the variation of k_s and h_c with temperature. A program was written to perform this calculation for prescribed values of initial temperature and pipe size; this was repeated for decreasing values of the time step δt until no further change in the freezing time was obtained.

5. RESULTS

Two investigations were carried out. Firstly, freezing times were calculated for four pipe diameters and various initial water temperatures and, secondly, the effect of the pipe size, jacket length and coolant temperature on the limiting temperature was investigated. The model was manipulated to develop a method of scaling the effect of convection on freezing.

5.1. *Predicted freezing times*

The freezing times were initially calculated for water at 0°C in pipes with diameters 100 mm, 150 mm, 200 mm and 250 mm with the following parameters: $T_c = -196$°C (liquid nitrogen) and $R_w^o/R_w^i = 1.1$.

The difference between the experimental results (extrapolated for 0°C from the values at higher temperatures) and these predictions were positive and increased with the pipe diameter. These offsets were attributed to the time taken to fill the jacket with nitrogen and were assumed to be approximately constant for a particular combination of pipe and jacket. This is an approximation because part of the offset is probably due to the assumption of a steady temperature profile in the ice; the transient profile is roughly linear and therefore the heat conducted through the ice is lower. During rapid freezing (i.e. in small diameter pipes with a low initial temperature), assuming a steady profile will underpredict the freezing time. The extent of this is difficult to quantify but it should be partially counteracted by the fact that the filling time will increase slightly with increasing water temperature due to the high boil off rates while filling the jacket.

The offsets were added into the following calculations for freezing time in water at temperatures above 0°C. The freezing times were calculated at 5°C intervals using the following parameters: $H = 4 \times R_w$ (ratio of jacket length to pipe diameter 2:1), $A_h = 1$ (neck at bottom of jacket) and $A_h = 1/2$ (neck at centre of jacket).

The resulting freeze times are shown in Fig. 3. Two sets of predicted freeze times were obtained for each pipe diameter; those obtained with $A_h = 1/2$ are shown by a dashed line and those for $A_h = 1$ with a solid line. The experimental data are included as symbols. Comparison between the two sets of data shows that the general behaviour is predicted well.

For a pipe diameter of 200 mm and above, the experimental data match the predictions obtained with the neck at the midpoint of the jacket ($A_h = 1/2$) while in the smaller pipes, setting the neck position at the bottom of the jacket give better agreement. The greater effect of convection in the larger pipes may be due to the flow becoming turbulent and increasing heat transfer.

The results for the 100 mm diameter pipe obtained with ($A_h = 1$) show good agreement below 35°C. The predicted limiting temperature is between 40 and 45°C compared to the experimental value of just over 45°C and the predicted freezing time at 44°C is higher than the experimental result. This may possibly be due to a decay in the water temperature during the experiments at 44 and 45°C; a heater and temperature controller was located in the tank below the pipe, however, it is possible that the temperature was not maintained effectively in the freezing zone. This would reduce the freezing time and increase the limiting temperature. More detailed numerical predictions of the temperature and velocity distributions obtained by Keary [11] demonstrated that the bulk temperature decreases during freezing in small diameter pipes and that this becomes less important in larger diameter pipes.

5.2. *Parametric study of limiting temperature*

From the preceding analysis, an indication of the highest value of initial water temperature that can be frozen was obtained for each pipe size considered. This method of predicting the maximum temperature uses the heat transfer coefficient derived for laminar natural convection driven by a cooled flat plate. Near the limiting temperature the assumptions made in the model derivation will become increasingly inaccurate, especially in terms of neck migration and turbulent flow. In spite of this, it was felt that an analysis of the factors which affect the limiting temperature would yield valuable information.

The plug will eventually form a solid cylinder of ice, sealing the pipe, if the rate of interface movement is less than zero for all values of interface radius between the pipe radius and zero. Therefore, considering equation (3), the first term in the parenthesis on the right-hand side of the equation must always be less than the second term. This first term is independent of the plug radius and for a given water temperature and pipe size remains constant throughout the freeze. The second term varies with the interface radius; as the radius decreases, the denominator of this term increases then decreases to equal zero when the radius is zero. There is maximum thermal resistance between the interface and coolant at an intermediate point in the freeze and therefore a related minimum heat flux. The interface radius at which the freezing rate is a minimum is given by equation (10).

$$R_i^{crit} = R_w^i e^{-1+Ik_s} \quad (10)$$

Therefore the necessary condition for the plug to completely seal the pipe can be simplified to equation (11).

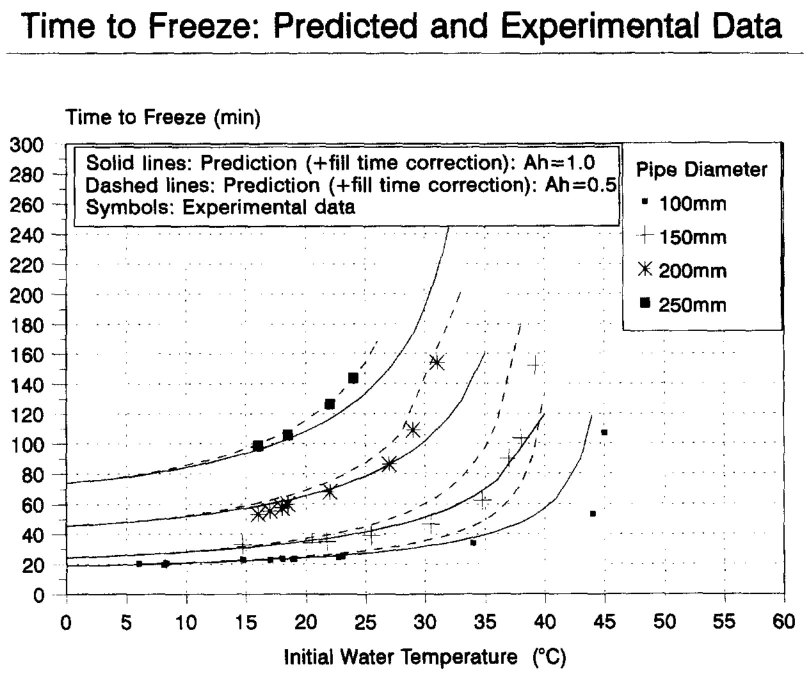

Fig. 3. Freezing times predicted by analytical model: comparison with experimental results.

$$hT_b < \frac{k_s|T_c|}{R_w^o e^{-1} e^{Ik_s}} \tag{11}$$

The limiting temperature is the value for which the two terms are equal. With a constant heat transfer coefficient and low thermal resistance between the ice and the coolant, the limiting temperature would be inversely proportional to the pipe radius, as noted by Bowen *et al.* [3]. Using the expression for heat transfer coefficient from Bejan, the limiting temperature is given in equation (12).

$$T_b^{\lim} = \left[\frac{k_s|T_c|(A_h H)^{0.25}}{25.880 R_w^o e^{-1} e^{Ik_s}}\right]^{1/1.68} \tag{12}$$

Iteration is required to solve this equation due to the variation of k_s and I with temperature. The effect of varying the pipe diameter, jacket length and coolant temperature on the limiting temperature was studied using the parameters listed in Table 1 and the results are described in the following sections.

Effect of varying the pipe diameter. The limiting temperature was calculated using the parameters above and the results are plotted in Fig. 4.

Comparison with the experimental data shows that the experimental results drop off more rapidly with increasing pipe size than the predicted values. A possible cause of this may be the assumption of laminar natural convection. Transition from laminar to turbulent flow for natural convection is quantified by a critical Grashof number. The critical Grashof number of 1.3×10^9 for flow over a heated vertical plate (Bejan [9]) is included in Fig. 4 (based on the jacket length). This indicates that the experimental data for limiting temperature lie on a line defined by a Grashof number (using the full jacket length) of 2.1×10^9, suggesting that the limiting conditions are controlled by the flow becoming turbulent.

Below a critical diameter which lies in the range 100–150 mm, the limiting temperature predicted under laminar conditions is greater than that pre-

Table 1. Test conditions for limiting temperature study

	Test		
Parameter	Pipe diameter	Jacket length	Coolant temp.
R_w^o/R_w^i	1.1	1.1	1.1
T_c	−196°C	−196°C	−200–0°C
h_c	Calculated	Calculated	BIG
R_w^i (mm)	50, 75, 100, 125	50, 75, 100, 125	50, 75, 100, 125
A_h	1, 1/2	1	1
H/R_w^i	4	2–10	4

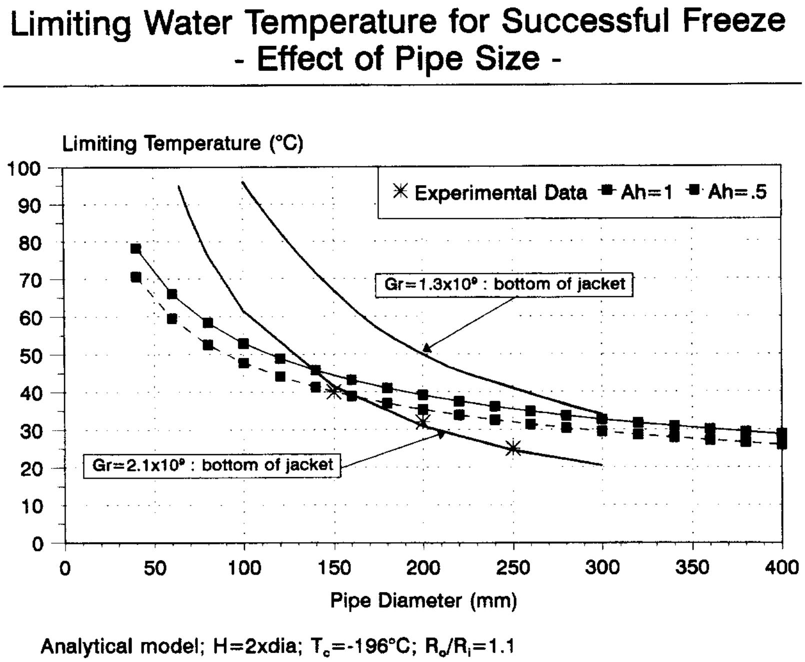

Fig. 4. Predicted limiting temperature with varying pipe diameters.

dicted by the value of Grashof number. Transition criteria should only be treated as estimates of order of magnitude, however, if the transition criterion ($Gr = 1.3 \times 10^9$) is applicable, this implies that freezing will stop when the boundary layer flow has become turbulent over approximately 85% of the jacket length.

Effect of varying the jacket length. The predicted relationship between the ratio of jacket length to pipe diameter and limiting temperature for four pipe sizes is plotted in Fig. 5. This shows an approximately linear increase in the limiting temperature with increasing jacket length. The effect is slightly greater in a 100 mm diameter pipe (≈ 15°C increase obtained by increasing the jacket length from one pipe diameter to five) than in a 250 mm diameter pipe (≈ 10°C over the same range).

This prediction assumed laminar flow conditions, however, using longer jackets will decrease the temperature at which the flow becomes turbulent and therefore, with the exception of very small diameter pipes, will decrease the limiting temperature. The existence of a laminar region and a turbulent region may cause the formation of multiple necks in the ice plug; if the plug closes off at two points, the water trapped in between will freeze rapidly and the resulting expansion may be sufficient to cause damage to the pipe.

Effect of varying the coolant temperature. For comparison between different coolant temperatures, it was necessary to ignore the thermal impedance between the coolant and the wall. The relationship between the limiting temperature and the coolant temperature is plotted in Fig. 6. This shows a roughly linear decrease in limiting temperature with increasing (decreasing magnitude) coolant temperature. The effect is greater in the 100 mm diameter pipe than in the large pipes. These results have practical implications for the use of higher temperature ('controlled temperature') freezing methods.

5.3. *Scaling the effect of natural convection on freezing*

One aim of this investigation was to develop an understanding of the processes involved in pipe freezing to make it possible to predict the behaviour under conditions which have not been studied in detail. For instance, a large number of experiments have been carried out looking at the effect of convection on freezing using a 100 mm diameter pipe (Burton [1] and Tavner [2]); until now, a way of scaling these results to predict the behaviour in larger pipes has not been available.

The effect of convection on freezing can be defined using the ratio of the convected flux (at the interface) to the conducted flux (removed through the ice layer). This is a maximum at the critical radius. If the thermal impedance between the ice and coolant is neglected (k_w, h_c = BIG), this ratio is given by equation (13).

$$\frac{Q_{\text{conv}}}{Q_{\text{cond}}} = \frac{R_w e^{-1} h T_b}{k_s |T_c|} \tag{13}$$

For a fixed coolant temperature, the conductivity of ice will not change and the product hT_bR_w provides a measure of the effect of convection on freezing (heat flow per unit length). This is defined as the "effective heat flux" (e.h.f.). Using the approximation for the heat transfer coefficient (equation (8)) and assuming that the jacket length is a constant multiple of the pipe

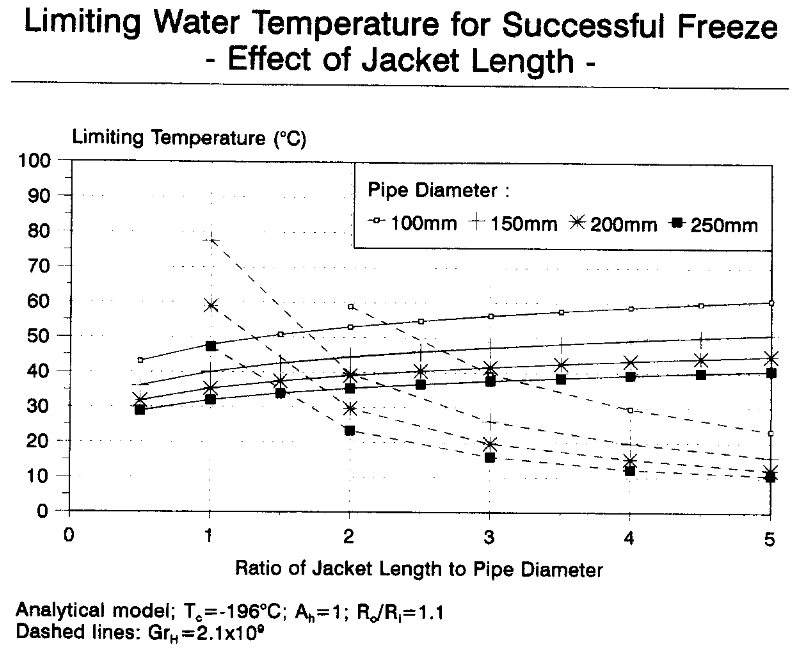

Fig. 5. Predicted limiting temperature with varying jacket length.

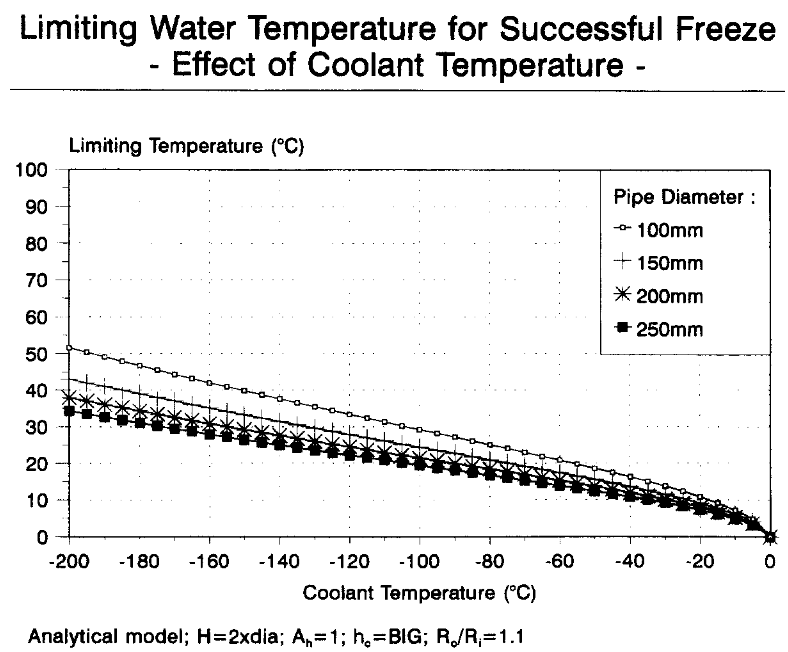

Fig. 6. Predicted limiting temperature with coolant temperature.

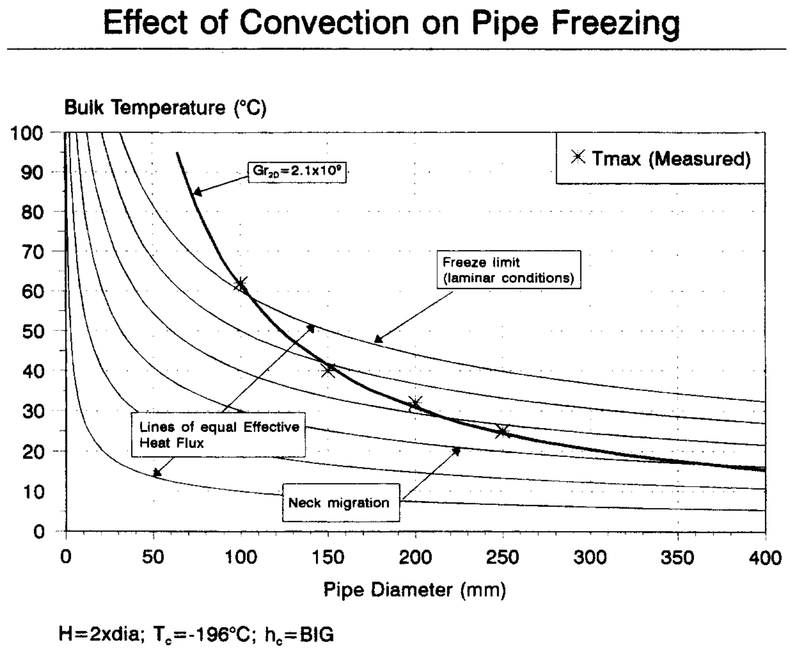

Fig. 7. Effect of convection on the freezing process.

radius, gives the following expression for the e.h.f. (equation (14)).

$$\text{e.h.f.} = hT_bR_w = KR_w^{0.75}T_b^{1.68} \qquad (14)$$

where

$$K = \left[25.880\left(A_h\frac{H}{R_w}\right)^{-0.25}\right]$$

To avoid introducing a new set of values (namely e.h.f.), equation (14) was simplified into a form which could be used to express similarity between different pipe sizes and water temperatures in terms of the effect of convection on freezing. This is given in equation (15).

$$T_bR_w^{0.446} = \text{constant} \Rightarrow \text{constant effective heat flux} \qquad (15)$$

Burton [1] and Tavner [2] noted that the effect of convection on freezing was negligible in a 100 mm diameter pipe at temperatures below 20°C. By using equation (15) the equivalent temperatures in other diameter pipes can be estimated, for example giving a cut-off temperature of 15°C in a 200 mm diameter pipe.

This approach was used to produce Fig. 7, showing the effect of convection on pipe freezing. Six lines of equal effective heat flux (e.h.f.) are plotted (shown as thin solid lines); these lines are defined by the values of e.h.f. corresponding to temperatures of 10, 20, 30, 40, 50 and 60°C in a 100 mm diameter pipe. The upper solid line indicates the limiting temperature predicted using equation (12). The thick line denotes Grashof number equal to 2.1×10^9 at the bottom of the jacket which fits the experimental data for the limiting water temperature.

In addition to the limitations imposed by the assumption of laminar natural convection, the application of this approach is also constrained by the assumption that the plug grows as a thickening cylinder. If the plug is symmetric around the neck, the assumption of one-dimensional conduction with zero axial conduction and the use of a flat plate heat transfer coefficient will be valid. Increasing asymmetry, most commonly linked to neck migration, decreases the validity of these assumptions. The main problem lies in neglecting axial heat conduction through the ice; this increases the heat flux removed through the ice from the interface. The effect on the heat transfer coefficient is less important, with the heat flux being over predicted as the water above the plug neck becomes increasingly isolated from the water below.

Neck migration occurs when the conditions above and below the plug neck are sufficiently different to affect the freezing rates. This is controlled by the effect of convection on the local freezing rate and can be characterised with a value of e.h.f. Burton noted that the plug neck moved up the plug during freezing when the water temperature was maintained at 30°C and no neck migration was noted at 20°C; this sets a limit on applying the model as shown by a dashed line in Fig. 7.

6. FURTHER MODEL DEVELOPMENT

The model could be developed further to include the effect of turbulence, by incorporating an empirical turbulent heat transfer coefficient. In order to develop a more accurate model for predicting the freezing times for relatively high temperature freezes, a more sophisticated, probably numerical, approach would include axial conduction and the transient temperature behaviour. In order to predict the neck migration, the full flow field must be modelled. Including a turbulence model would improve the accuracy for larger pipes and higher temperatures.

7. CONCLUSIONS

A solution for the problem of pipe freezing with natural convection has been derived. This provides an estimate of the freeze time and limiting temperature and can be applied to different pipe sizes, fluid temperatures, jacket lengths, cooling methods and fluids.

The predictions show the same general behaviour as that recorded in the experiments. The freeze times with superheat were much lower than the experimental values, primarily due to neglecting the time to fill the freezing jacket. The predicted values increased less rapidly with increasing temperature than the experimental results due to a combination of the assumptions of laminar convection and zero axial conduction in the ice.

The study indicated that the limiting temperature is controlled by laminar natural convection in pipes less than 100–150 mm in diameter; in larger pipes the flow conditions become turbulent (defined by a constant Grashof number). A criterion to scale the effect of convection on freezing between pipe sizes under laminar conditions was defined which proposed that a constant value of $T_b R_w^{0.446}$ implies a constant effect of convection on freezing.

REFERENCES

1. Burton, M. J., An experimental and numerical study of plug formation in vertical pipes during pipe freezing. Ph.D. thesis, University of Southampton, U.K., 1986.
2. Tavner, A. C. R., An experimental study of ice formation and convection during cryogenic pipe freezing. Ph.D. thesis, University of Southampton, U.K., 1992.
3. Bowen, R. J., Burton, M. J. and Smith, G. S., The effect of pipe diameter and pressure drop on the formation of ice plugs in pipelines. *Ninth International Heat Transfer Conference*. Jerusalem, Israel, 1990.
4. London, A. L. and Seban, R. A., Rate of ice formation. *Trans. ASME*. 1943, **65,** 771–778.
5. Luikov, A. V., Solidification of a cylinder. *Analytical Heat Diffusion Theory*. Academic Press, New York, 1968, pp. 452–453.
6. Shamsundar, N., Approximate calculation of multidimensional solidification by using conduction shape factors. *Trans. ASME J. Heat Transfer*, 1982, **104** nl, pp. 8–12.
7. Law, K. M., An investigation into cryogenic pipe freezing. Ph.D. thesis, Paisley College of Technology, Glasgow, U.K., 1989.
8. Lannoy, A., Problèmes thermiques posés par la congélation d'un fluide dans une tuyauterie. *Revue Géneral Thermique*, 1980, **220,** 311–320 (in French). Thermal problems involved in freezing a fluid in pipework. Translation from: British Library Document Supply Centre DSC 6196.3, 1990.
9. Bejan, A., *Convection Heat Transfer*. John Wiley and Sons, New York, U.S.A., 1984.
10. Flynn, T. M., Draper, J. W. and Roos, J. J. *Advances in Cryogenic Engineering*, 1962, **7,** 539–545.
11. Keary, A. C., A numerical study of solidification and natural convection during cryogenic pipe freezing. Ph.D. thesis, University of Southampton, U.K., 1995.